ISING MACHINES

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Outline

• Some properties of light
  • Superposition, Entanglement
  • Wave – Particle Duality
  • Classical versus Quantum Experiments

• Photonic Ising Machines
  • Multiplexed in Time – Poor Man’s Ising Machine
  • Multiplexed in Space – Spatial Light Modulation

• Chip-based Ising Computing Machines
  • Electronic oscillators
  • Magnetic oscillators
Properties of Light

- Polarization
- Color
  - Wavelength, Frequency
- Phase
- Spatial Modes
How does Quantum Computing work?

- Classical bits versus quantum qubits

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2 bits of information  
State of A and of B

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4 bits of information  
Superposition state

Waves interfere, we use them for computation

- Extend this to 3 qubits – 8 bits of information
- n-qubits will have \(2^n\) bits of information
How does Photonic Computing work?

- Double slit experiment – wave particle duality
- Analogy – massively parallel computation

(a) Particles

(b) Waves

(c) Particles as Waves
Testing a Single Photon Source

- Hong-Ou-Mandel Interferometer

Coincidence of counts shows a non-classical dip

HOM dip as a function of $T$
How does a laser work?

- Optical wave (photons) oscillating inside a resonant cavity

Stimulated Emission is in phase with the incoming photon.

A laser is Coherent. All the photons are locked in phase.

Can measure a coherence time.
How do we resolve photon numbers?

- Redistribution into different spatial or temporal bins

Pulses with 5 photons:

- Spatial redistribution method
- Temporal redistribution method

Diagram showing the redistribution process with detectors for each photon level.
Temporal redistribution

- Average power reduced by half for each circulation
- Synchronized detection using gating at GAPD

![Diagram of Temporal Redistribution](image)
Detection probabilities

- Decreasing detection probabilities with subsequent redistribution
Predicting detections

- If it is classical, we can predict patterns
  - 10..., 11
  - 100..., 101, 110, 111...
  - 1000, 1001, 1010, 1011 ... etc
The Ising Model

- Popular model in Statistical Physics
- Phase Transition
- Percolation Theory
- Ferromagnetic vs Antiferromagnetic

\[ H_{\text{Ising}} = -\frac{1}{2} \sum_{mn}^{N} J_{mn} \sigma_m \sigma_n. \]

\[
I = \begin{cases} 
> 0 & \text{ferromagnetic} \\
< 0 & \text{antiferromagnetic} 
\end{cases}
\]
Bloch Wall

- Spins will orient themselves based on the minimum energy configuration.
- Costs less energy to create a Wall than to flip one spin.
The Poor Man’s Ising Machine

- Mach Zender Modulator with optoelectronic feedback
The Optical-Electronic-Optical Model

\[
x_n[k + 1] = \cos^2(f_n[k] - \pi/4 + \zeta_n[k]) - \frac{1}{2}.
\]

\[
f_n[k] = \alpha x_n[k] + \beta \sum_m J_{mn} x_m[k].
\]

\[
\sigma_n = \text{sign}(x_n[k]).
\]

- Self bias term \( \alpha \)
- Coupling coefficient \( \beta \)
- Weights between spins \( J \)
- What does the optics do?
  - Nonlinear function – \( \cos^2 \)
MZM Transfer Function

\[ I_{out}(t) = T_{mod} \frac{I_{in}}{2} \left[ 1 + \cos \left( \frac{\pi}{V_{\pi}} V(t) - \phi \right) \right] \]

- Find the correct bias point, so we have two solutions - bifurcation
Pitchfork Bifurcation

- Nonlinear function has two solutions
- Uncoupled spins
Pitchfork Bifurcation

- With 100 spins
- Tune $\alpha$ to observe the bifurcation
Square Lattice

- 10 x 10 lattice, $\alpha = 0.25$, $\beta = 0.29$
- Can observe domain walls where spins are aligned up
- Lowest energy for the checkerboard pattern
The Poor Man’s Ising Machine

- Not quite Poor….about $10,000
The Poor Man’s Ising Machine

- It is actually a little more complicated
- Needed an optical isolator, a polarization controller, and an optical amplifier
16 x 16 spin lattice

- Nearest Neighbours on a square lattice
- Gautham and Parth, IIT Madras
Spin lattice (16x16, 24x24)
24 x 24 spin lattice

- Discontinuities are under investigation
- Gautham and Parth, IIT Madras
How well does it scale?

- Photonic Chips, NUS, Singapore
Spatial Light Ising Machine (SLIM)

Large-Scale Photonic Ising Machine by Spatial Light Modulation
PHYSICAL REVIEW LETTERS 122, 213902 (2019)
D. Pierangeli,¹,²,* G. Marcucci,¹,² and C. Conti¹,²

- Optical beams have spatial divergence
- Interactions are between different rays - phase
Understanding Phase

• Different rays of light accumulate a different phase based on their propagation distance from the source
• Think of diffraction in 2-D, you will see rings
Fourier Optics

- The 4-f system
- Every optical ray in 2-D interacts with every other ray
- A lens in 2-D acts as a Fourier Transformer, gives spatial frequency content
Spatial Filters

• Easy to design high pass, low pass and vertical pass filters
Large-Scale Photonic Ising Machine by Spatial Light Modulation
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- Amplitude mask
- Phase plane (SLM)
- Camera
- Feedback

Far field intensity
\[ I(x) = |E(x)|^2 = \sum_{jh} \xi_j \xi_h \sigma_j \sigma_h \delta_W^2(x) e^{2iW(h-j)} \]

Fourier transform of a rectangle
\[ \delta_W(x) = \sin(Wx)/(Wx) \]
SLIM Algorithm

What are its limitations?

- We need $\Phi(x,y)$
- Write an image to the SLM
- Read 1000 x 1000 pixels off a camera

- Computer-generated holographic optical tweezer arrays
- Dufresne et al, Rev. Sci. Instr., 72, 2001
Convergence Issues

• We lose phase information with a camera
• SLM is slow and resets every so often
• 500 iterations takes 1 hour
Chip based Ising Computing Machine

- CICM oscillators can be optical, magnetic or electronic
The “free” magnet aligns itself towards a preferred direction determined by the injected current.

The “free” magnet oscillates about an effective field (analogous to a top precessing in a gravitational field).
Coupled Nano Oscillators

- Arrays of oscillators can lock together
- How do we change the coupling?

Nano-Patterned Coupled Spin Torque Nano Oscillator (STNO) Arrays – a Potentially Disruptive Multipurpose Nanotechnology

Mircea R. Stan, Mehdi Kabir
ECE Dept., University of Virginia
Charlottesville, VA, USA

Jiwei Lu, Stuart Wolf
MSE Dept., University of Virginia
Charlottesville, VA, USA
State of the Art

- Can do weighted oscillator networks – vowel recognition
- Romera et al, Nature 563, 2018
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