

47-779. QUANTUM INTEGER PROGRAMMING

Mini-1, Fall 2020

Room: Zoom Online **Time:** Tuesday and Thursday 5:20pm-7:10pm

Instructors:

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Office Hours: Post your questions in the forum provided for this purpose on Canvas. This course will be conducted online.

Objectives: This course is primarily designed for graduate students (and advanced undergraduates) interested in integer programming (with non-linear objective functions) and the potential of near-term quantum and quantum-inspired computing for solving combinatorial optimization problems. By the end of the semester, someone enrolled in this course should be able to:

- Appreciate the current status of quantum computing and its potential use for integer programming
- Access and use quantum computing resources (such as D-Wave Quantum Annealers)
- Set up a given integer program to be solved with quantum computing
- Work in groups collaboratively on a state-of-the-art project involving applications of quantum computing and integer programming

This course is not going to focus on the following topics:

- Quantum Gates and Circuits
- Computational complexity theory
- Quantum Information Theory
- Analysis of speedup using differential geometry, algebraic topology, etc.

Prerequisite classes and capabilities: Although this class has no explicit prerequisites we consider a list of recommended topics and skills that the student should feel comfortable with. An undergraduate-level understanding of probability, calculus, statistics, graph theory, algorithms, and linear algebra is assumed. Knowledge of linear and integer programming will be useful for this course. Programming skills are strongly recommended. Basic concepts in physics are recommended but lack of prior knowledge is not an issue as pertinent ones will be covered in the lectures. No particular knowledge in quantum mechanics or algebraic geometry is required.

Students with backgrounds in operations research, industrial engineering, chemical engineering, electrical engineering, physics, computer science, or applied mathematics are strongly encouraged to consider taking this course.

Tentative Course Outline:

- Part 1 - Integer programming (classical methods)** 1 week
- █ Integer Programming basics [1].
 - █ Cutting plane theory and relaxations [1].
 - █ Introduction to Test Sets [2, 3, 4, 5].
 - █ Gröbner basis [6, 7, 8].
 - █ Graver basis [9].
- Part 2 - Ising, QUBO** 1 week
- █ Ising model basics [10, 11, 12].
 - █ Simulated Annealing [13, 14].
 - █ Markov-chain Monte Carlo methods [15, 16, 17, 18].
 - █ Benchmarking classical methods [19, 20].
 - █ Formulating combinatorial problems as QUBOs [21].
- Part 3 - GAMA: Graver Augmented Multiseed algorithm** 1 week
- █ GAMA [22].
 - █ Applications: Portfolio Optimization [22], Cancer Genomics [23]
 - █ Quantum Inspired: Quadratic (Semi-)Assignment Problem [24].
- Part 4 - Quantum methods for solving Ising/QUBO** 1 week
- █ AQC, Quantum Annealing and D-Wave [25, 26, 27, 28, 29, 30, 31].
 - █ QAOA: Quantum Approximate Optimization Algorithm [32, 33, 34].
- Part 5 - Hardware for solving Ising/QUBO** 1 week
- █ Graphical Processing Units [35, 36, 37].
 - █ Tensor Processing Units [38].
 - █ Complementary metal-oxide-semiconductors (CMOS) [39].
 - █ Digital Annealers [40].
 - █ Oscillator Based Computing [41, 42].
 - █ Coherent Ising Machines [43, 44, 45, 46, 47, 48].
- Part 6 - Other topics and project presentations** 1 week
- █ Compiling
 - █ Quantum Annealing [49, 50].
 - █ Gate-based Noisy Intermediate Scale Quantum (NISQ) devices [51].
 - █ Adiabatic Quantum Computing and Algebraic Geometry
 - █ Minimizing Polynomial Functions [52].
 - █ Prime Factorization [53].

Grading Policy: Weekly quizzes and scribe (50%), Final Group Project (50%).

- Each lecture will have a short quiz to evaluate the concepts covered in a previous class. The two worst quizzes won't be counted towards the final grade.
- Since this is a new course, we will need your help in compiling the lecture notes by signing up for scribe duties. Each student will complete the scribe duties of one lecture and this will replace one quiz grade. The notes will be written via Overleaf and The sign-up sheet can be found in Canvas.
- The group final project will require the implementation and solution of an integer programming instance. This final project deliverable is a report and a presentation.

Project description: The final project accounts for 50% of the total grade. This project will be completed in groups of 2-4 students and will reflect the understanding of the students of the material covered in the lecture. The components of this project are the following

- Identify a problem that can be posed as an Integer Program. Discuss the importance of this problem.
- Solve instances of the identified problem using classical tools. Identify which are the sources of complexity while solving this problem.
- Model the problem as a Quadratic Unconstrained Binary Optimization (QUBO). Verify that the reformulation of the problem is valid, in the sense that it represents the original problem.
- Solve the resulting QUBO using non-conventional methods, e.g. Quantum Annealing, QAOA, simulated annealing in GPUs/TPUs, etc. Compare at least two different methods.
- Write a report outlining the different approaches used and highlighting the knowledge obtained while developing the project.
- Hold a final presentation in front of the class reporting the findings of the project.

Important Dates:

No Final Exam. Presentations in weeks 6-7.

Course Policy:

- Auditing students are encouraged to participate actively in the lectures.
- Regular attendance is essential and expected.
- Please sign up using Canvas.

Highlights: The specific skills that students will gain that will make them “quantum ready” for industry or further academic research in this course are:

1. Classical
 - (a) Given a practical problem (from supply chain or physics or anything else), formulate it as a non-linear integer program. We will provide a few practical problems, but we encourage you to suggest one that you are already working on or are interested in.
 - (b) Solve such formulations via classical solvers.
2. Quantum
 - (a) Reformulate the problem to be “quantum ready” by making it in the form of a QUBO.
 - (b) Solve the QUBO “brute force” through D-Wave or IBM (via QAOA).
3. Hybrid Quantum-Classical
 - (a) Reformulate the problem again in the form suitable for GAMA.
 - (b) Solve GAMA compatible formulation via D-Wave and/or via QAOA.

USRA Collaboration

1. Access to D-Wave systems might be available via written proposals to the University Space Research Association (USRA). See <https://riacs.usra.edu/quantum/rfp> for terms and conditions. The course will discuss proposal preparation.
2. Students of this course are encouraged to apply to the Feynman Academy Internship program <https://riacs.usra.edu/quantum/qacademy> that sponsors research projects at NASA Ames Research Center.

Amazon Braket Collaboration

1. Access to D-Wave, Rigetti and ionQ have been made available by Amazon Braket.
2. Students will be provided individual sub-accounts for use with pre-funded amounts to access the quantum machines.

Academic Honesty: Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Any form of plagiarism can earn you a failing grade for the entire course. For more information you can refer to [CMU's policies on academic integrity](#). When in doubt, add a citation.

Casual References: There is no single text book for the course. This is a short list of various interesting and useful books that will be mentioned during the course. You need to consult them occasionally.

- Georges Irfah, *The Universal History of Computing*, John Wiley & Sons, 2001.
- A. Das and B.K. Chakrabarti (Eds.). *Quantum Annealing and Related Optimization Methods*, Springer-Verlag, 2005.
- Eleanor G. Rieffel and Wolfgang H. Polak, *Quantum Computing: A Gentle Introduction*, MIT Press, 2011.
- Richard J. Lipton and Kenneth W. Regan, *Quantum Algorithms via Linear Algebra. A Primer*, MIT Press, 2014.

Student Resources: If you have a disability and require accommodations, please contact Catherine Getchell, Director of Disability Resources, 412-268-6121, getchell@cmu.edu. If you have an accommodations letter from the Disability Resources office, we encourage you to discuss your accommodations and needs with us as early in the semester as possible. We will work with you to ensure that accommodations are provided as appropriate.

As a student, you may experience a range of challenges that can interfere with learning, such as strained relationships, increased anxiety, substance use, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may diminish your academic performance and/or reduce your ability to participate in daily activities. CMU services are available, and treatment does work. You can learn more about confidential mental health services available on campus at: <http://www.cmu.edu/counseling/>. Support is always available (24/7) from Counseling and Psychological Services: 412-268-2922.

References

- [1] Michele Conforti, Gérard Cornuéjols, Giacomo Zambelli, et al. *Integer programming*, volume 271. Springer, 2014. [2](#)
- [2] Robert Weismantel. Test sets of integer programs. *Mathematical Methods of Operations Research*, 47(1):1–37, 1998. [2](#)
- [3] Bernd Sturmfels. *Grobner bases and convex polytopes*, volume 8. American Mathematical Soc., 1996. [2](#)
- [4] Sridhar R Tayur, Rekha R Thomas, and NR Natraj. An algebraic geometry algorithm for scheduling in presence of setups and correlated demands. *Mathematical Programming*, 69(1-3):369–401, 1995. [2](#)
- [5] Jesús A De Loera, Raymond Hemmecke, and Matthias Köppe. *Algebraic and geometric ideas in the theory of discrete optimization*. SIAM, 2012. [2](#)

- [6] Dimitris Bertsimas, Georgia Perakis, and Sridhar Tayur. A new algebraic geometry algorithm for integer programming. *Management Science*, 46(7):999–1008, 2000. 2
- [7] Pasqualina Conti and Carlo Traverso. Buchberger algorithm and integer programming. In *International Symposium on Applied Algebra, Algebraic Algorithms, and Error-Correcting Codes*, pages 130–139. Springer, 1991. 2
- [8] Serkan Hoşten and Bernd Sturmfels. Grin: An implementation of gröbner bases for integer programming. In *International Conference on Integer Programming and Combinatorial Optimization*, pages 267–276. Springer, 1995. 2
- [9] Raymond Hemmecke, Shmuel Onn, and Robert Weismantel. A polynomial oracle-time algorithm for convex integer minimization. *Mathematical Programming*, 126(1):97–117, 2011. 2
- [10] Stephen G Brush. History of the Lenz-Ising model. *Reviews of Modern Physics*, 39(4):883, 1967. 2
- [11] David Sherrington and Scott Kirkpatrick. Solvable model of a spin-glass. *Phys. Rev. Lett.*, 35:1792–1796, Dec 1975. doi: 10.1103/PhysRevLett.35.1792. URL <https://link.aps.org/doi/10.1103/PhysRevLett.35.1792>. 2
- [12] P. Ray, B. K. Chakrabarti, and Arunava Chakrabarti. Sherrington-kirkpatrick model in a transverse field: Absence of replica symmetry breaking due to quantum fluctuations. *Phys. Rev. B*, 39:11828–11832, Jun 1989. doi: 10.1103/PhysRevB.39.11828. URL <https://link.aps.org/doi/10.1103/PhysRevB.39.11828>. 2
- [13] Scott Kirkpatrick, C Daniel Gelatt, and Mario P Vecchi. Optimization by simulated annealing. *Science*, 220(4598):671–680, 1983. 2
- [14] Christos Koulamas, SR Antony, and R Jaen. A survey of simulated annealing applications to operations research problems. *Omega*, 22(1):41–56, 1994. 2
- [15] Nicholas Metropolis, Arianna W Rosenbluth, Marshall N Rosenbluth, Augusta H Teller, and Edward Teller. Equation of state calculations by fast computing machines. *The Journal of Chemical Physics*, 21(6):1087–1092, 1953. 2
- [16] Alfred B Bortz, Malvin H Kalos, and Joel L Lebowitz. A new algorithm for monte carlo simulation of ising spin systems. *Journal of Computational Physics*, 17(1):10–18, 1975. 2
- [17] Matthias Troyer and Uwe-Jens Wiese. Computational complexity and fundamental limitations to fermionic quantum monte carlo simulations. *Physical Review Letters*, 94(17):170201, 2005. 2
- [18] A Peter Young, Sergey Knysh, and Vadim N Smelyanskiy. Size dependence of the minimum excitation gap in the quantum adiabatic algorithm. *Physical Review Letters*, 101(17):170503, 2008. 2
- [19] Iain Dunning, Swati Gupta, and John Silberholz. What works best when? a systematic evaluation of heuristics for max-cut and qubo. *INFORMS Journal on Computing*, 30(3):608–624, 2018. 2
- [20] Carleton Coffrin, Harsha Nagarajan, and Russell Bent. Evaluating ising processing units with integer programming. In *International Conference on Integration of Constraint Programming, Artificial Intelligence, and Operations Research*, pages 163–181. Springer, 2019. 2
- [21] Andrew Lucas. Ising formulations of many np problems. *Frontiers in Physics*, 2:5, 2014. 2
- [22] Hedayat Alghassi, Raouf Dridi, and Sridhar Tayur. Graver bases via quantum annealing with application to non-linear integer programs. *arXiv preprint arXiv:1902.04215*, 2019. 2
- [23] Hedayat Alghassi, Raouf Dridi, A Gordon Robertson, and Sridhar Tayur. Quantum and quantum-inspired methods for de novo discovery of altered cancer pathways. *bioRxiv*, page 845719, 2019. 2

- [24] Hedayat Alghassi, Raouf Dridi, and Sridhar Tayur. Gama: A novel algorithm for non-convex integer programs. *arXiv preprint arXiv:1907.10930*, 2019. 2
- [25] Catherine C McGeoch. Theory versus practice in annealing-based quantum computing. *Theoretical Computer Science*, 816:169–183, 2020. 2
- [26] Tameem Albash and Daniel A Lidar. Adiabatic quantum computation. *Reviews of Modern Physics*, 90(1):015002, 2018. 2
- [27] Arnab Das and Bikas K. Chakrabarti. Colloquium: Quantum annealing and analog quantum computation. *Rev. Mod. Phys.*, 80:1061–1081, Sep 2008. doi: 10.1103/RevModPhys.80.1061. URL <https://link.aps.org/doi/10.1103/RevModPhys.80.1061>. 2
- [28] Giuseppe E Santoro and Erio Tosatti. Optimization using quantum mechanics: quantum annealing through adiabatic evolution. *Journal of Physics A: Mathematical and General*, 39(36):R393, 2006. 2
- [29] Edward Farhi, Jeffrey Goldstone, Sam Gutmann, Joshua Lapan, Andrew Lundgren, and Daniel Preda. A quantum adiabatic evolution algorithm applied to random instances of an np-complete problem. *Science*, 292(5516):472–475, 2001. 2
- [30] Tadashi Kadowaki and Hidetoshi Nishimori. Quantum annealing in the transverse ising model. *Physical Review E*, 58(5):5355, 1998. 2
- [31] M. W. Johnson, M. H. S. Amin, S. Gildert, T. Lanting, F. Hamze, N. Dickson, R. Harris, A. J. Berkley, J. Johansson, P. Bunyk, E. M. Chapple, C. Enderud, J. P. Hilton, K. Karimi, E. Ladizinsky, N. Ladizinsky, T. Oh, I. Perminov, C. Rich, M. C. Thom, E. Tolkacheva, C. J. S. Truncik, S. Uchaikin, J. Wang, B. Wilson, and G. Rose. Quantum annealing with manufactured spins. *Nature*, 473(7346):194–198, May 2011. ISSN 0028-0836. doi: 10.1038/nature10012. URL <https://www.nature.com/nature/journal/v473/n7346/full/nature10012.html>. 2
- [32] Edward Farhi, Jeffrey Goldstone, and Sam Gutmann. A quantum approximate optimization algorithm. *arXiv preprint arXiv:1411.4028*, 2014. 2
- [33] Stuart Hadfield, Zhihui Wang, Eleanor G Rieffel, Bryan O’Gorman, Davide Venturelli, and Rupak Biswas. Quantum approximate optimization with hard and soft constraints. In *Proceedings of the Second International Workshop on Post Moores Era Supercomputing*, pages 15–21, 2017. 2
- [34] Stuart Hadfield, Zhihui Wang, Bryan O’Gorman, Eleanor G Rieffel, Davide Venturelli, and Rupak Biswas. From the quantum approximate optimization algorithm to a quantum alternating operator ansatz. *Algorithms*, 12(2):34, 2019. 2
- [35] A. Yavorsky, L. A. Markovich, E. A. Polyakov, and A. N. Rubtsov. Highly parallel algorithm for the ising ground state searching problem, 2019. 2
- [36] Chase Cook, Hengyang Zhao, Takashi Sato, Masayuki Hiromoto, and Sheldon X-D Tan. Gpu-based ising computing for solving max-cut combinatorial optimization problems. *Integration*, 69:335–344, 2019. 2
- [37] Joshua Romero, Mauro Bisson, Massimiliano Fatica, and Massimo Bernaschi. A performance study of the 2d ising model on gpus. *arXiv preprint arXiv:1906.06297*, 2019. 2
- [38] Kun Yang, Yi-Fan Chen, Georgios Roumpos, Chris Colby, and John Anderson. High performance monte carlo simulation of ising model on tpu clusters. In *Proceedings of the International Conference for High Performance Computing, Networking, Storage and Analysis*, pages 1–15, 2019. 2
- [39] Masanao Yamaoka, Chihiro Yoshimura, Masato Hayashi, Takuya Okuyama, Hidetaka Aoki, and Hiroyuki Mizuno. A 20k-spin ising chip to solve combinatorial optimization problems with cmos annealing. *IEEE Journal of Solid-State Circuits*, 51(1):303–309, 2015. 2

- [40] Maliheh Aramon, Gili Rosenberg, Elisabetta Valiante, Toshiyuki Miyazawa, Hirotaka Tamura, and Helmut Katzgrabeer. Physics-inspired optimization for quadratic unconstrained problems using a digital annealer. *Frontiers in Physics*, 7:48, 2019. [2](#)
- [41] Jeffrey Chou, Suraj Bramhavar, Siddhartha Ghosh, and William Herzog. Analog coupled oscillator based weighted ising machine. *Scientific Reports*, 9(1):1–10, 2019. [2](#)
- [42] Tianshi Wang and Jaijeet Roychowdhury. Oim: Oscillator-based ising machines for solving combinatorial optimisation problems. In *International Conference on Unconventional Computation and Natural Computation*, pages 232–256. Springer, 2019. [2](#)
- [43] Charles Roques-Carmes, Yichen Shen, Cristian Zanoci, Mihika Prabhu, Fadi Atieh, Li Jing, Tena Dubček, Chenkai Mao, Miles R Johnson, Vladimir Čeperić, et al. Heuristic recurrent algorithms for photonic ising machines. *Nature Communications*, 11(1):1–8, 2020. [2](#)
- [44] Takahiro Inagaki, Yoshitaka Haribara, Koji Igarashi, Tomohiro Sonobe, Shuhei Tamate, Toshimori Honjo, Alireza Marandi, Peter L McMahon, Takeshi Umeki, Koji Enbutsu, et al. A coherent ising machine for 2000-node optimization problems. *Science*, 354(6312):603–606, 2016. [2](#)
- [45] Andrew D King, William Bernoudy, James King, Andrew J Berkley, and Trevor Lanting. Emulating the coherent ising machine with a mean-field algorithm. *arXiv preprint arXiv:1806.08422*, 2018. [2](#)
- [46] Ryan Hamerly, Takahiro Inagaki, Peter L McMahon, Davide Venturelli, Alireza Marandi, Tatsuhiro Onodera, Edwin Ng, Carsten Langrock, Kensuke Inaba, Toshimori Honjo, et al. Experimental investigation of performance differences between coherent ising machines and a quantum annealer. *Science advances*, 5(5):eaau0823, 2019. [2](#)
- [47] Egor S Tiunov, Alexander E Ulanov, and AI Lvovsky. Annealing by simulating the coherent ising machine. *Optics express*, 27(7):10288–10295, 2019. [2](#)
- [48] Peter L McMahon, Alireza Marandi, Yoshitaka Haribara, Ryan Hamerly, Carsten Langrock, Shuhei Tamate, Takahiro Inagaki, Hiroki Takesue, Shoko Utsunomiya, Kazuyuki Aihara, et al. A fully programmable 100-spin coherent ising machine with all-to-all connections. *Science*, 354(6312):614–617, 2016. [2](#)
- [49] David E Bernal, Kyle EC Booth, Raouf Dridi, Hedayat Alghassi, Sridhar Tayur, and Davide Venturelli. Integer programming techniques for minor-embedding in quantum annealers. *arXiv preprint arXiv:1912.08314*, 2019. [2](#)
- [50] Raouf Dridi, Hedayat Alghassi, and Sridhar Tayur. A novel algebraic geometry compiling framework for adiabatic quantum computations. *arXiv preprint arXiv:1810.01440*, 2018. [2](#)
- [51] Raouf Dridi, Hedayat Alghassi, and Sridhar Tayur. Knuth-bendix completion algorithm and shuffle algebras for compiling nisq circuits. *arXiv preprint arXiv:1905.00129*, 2019. [2](#)
- [52] Raouf Dridi, Hedayat Alghassi, and Sridhar Tayur. Minimizing polynomial functions on quantum computers. *arXiv preprint arXiv:1903.08270*, 2019. [2](#)
- [53] Raouf Dridi and Hedayat Alghassi. Prime factorization using quantum annealing and computational algebraic geometry. *Scientific reports*, 7:43048, 2017. [2](#)